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Generalized Groups

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الزمر المعممة

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ABSTRACT

The aim of this paper is to generalize the concept of a group where the triple (G, E, ★) represents a generalized group if G is a nonempty set, E represents an equivalence relation defined on G and the operation ★ is a function of G×G satisfying some conditions. The paper also reveals some concepts and provides some examples related to generalized groups.

الملخص

يهدف البحث إلى تعميم مفهوم الزمرة، حيث يمثل الثلاثي (G,E,★) زمرة معممة إذا كانت G مجموعة غير خالية و E تمثل علاقة تكافؤ معرفة على G والعملية ★ دالة على G×G تحقق بعض الشروط. كما يكشف البحث عن بعض المفاهيم ويقدم بعض الأمثلة المرتبطة بالزمر المعممة.

KEYWORDS
الكلمات المفتاحية
Generalized subgroup, generalized semigroup, equivalence relation
الزمرة الجزئية المعممة، شبه الزمرة المعممة، علاقة التكافؤ

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1. Introduction

A generalized group has a deep physical background in the unified gauge theory, see (Molaei, 2000). The concept of equivalence relations will be used to generalize the concept of a group which has wide applications in our life. The aim of this work is to introduce the concepts of generalized groups, generalized subgroups, and generalized group homomorphisms.

This work will open a new area for scientific research. Indeed, the concept that had been generalized in this work will be the beginning of a lot of scientific research based on this generalized concept.

Historically, there have been several attempts to generalize the concept of group. Clifford characterized the semigroups that admit relative inverse (Clifford, 1941). A semigroup $(S,\star)$ is called a *semigroup admit relative inverses* if for any $x \in S$ there exist $e, y \in S$ such that $e \star x = x \star e = x$ and $x \star y = y \star x = e$. Later, these semigroups were called Clifford semigroups.

Molaei introduced generalized groups, as a class of algebras of interest in physics (Molaei, 1999). A semigroup $(S,\star)$ is called a *generalized group* if for any $x \in S$ there exist unique elements $e(x), x^{-1} \in S$ such that $e(x) \star x = x \star e(x) = x$ and $x \star x^{-1} = x^{-1} \star x = e(x)$. We call these semigroups Molaei generalized groups.

It is known that Molaei generalized groups are tools for constructions in unified geometric theory and electroweak theory. Araújo and Konieczny showed that Molaei generalized groups are the completely simple semigroups (Araújo and Konieczny, 2004; see also Akinmoyewa, 2009).

In this work, we generalize the concept of group using different meanings from Vagner (Vagner, 1952) generalized groups (Santilli, 1979), Clifford semigroups (Clifford, 1941; Clifford and Preston, 1961) and Molaei generalized groups (Molaei 1999 and 2000). Molaei generalized group will be a special case of generalized group which has been introduced in this work.

This paper is organized as follows:

- <u>In Section 2:</u> We begin by generalize the concept of a group as follows. Roughly speaking, a triple $(G, E,\star)$ is called a generalized group if G is a nonempty set, E is an equivalence relation on G, and $\star$ is a function on $G \times G$ that satisfies some conditions. New concepts are introduced as well as new examples of generalized groups. In addition, we investigate if some results that are true in classical group theory are also true in generalized groups.
- <u>In Section 3</u>: We continue our discussion of the basic properties of generalized groups with special attention to generalized subgroups.
- <u>In Section 4:</u> We consider one of the most important fundamental ideas of algebra, homomorphisms of generalized groups.

2. Generalized Groups

In this section, we generalize the classical definition of a group. Throughout this work, we consider a triple of the form $(G, E,\star)$, where G is an arbitrary nonempty set, E is an equivalence relation on G, and $\star$ is a function defined from $G \times G$ to a set X containing G.

The following definition widens the scope of groups and preserves the classical ones as special cases.

2.1. Definition:

A triple $(G, E,\star)$ is a *generalized group* if it satisfies the following conditions:

i. Closure: $x \star y \in G$ for any $x, y \in G$.
ii. Associativity: $x \star (y \star z) = (x \star y) \star z$ for any $x, y, z \in G$.
iii. Identity: for any $t \in G$, there is $e(t) \in [t]$ such that $e(t) \star x = x \star e(t) = x$ for any $x \in [t]$.
iv. Inverses: for any $t \in G$ and $x \in [t]$ there is $x^{-1} \in [t]$ such that $x \star x^{-1} = e(t)$.

Where $[t]$ denotes the equivalence class that contains $t \in G$.

A generalized group $(G, E,\star)$ with E begin the universal equivalence relation on G; *i.e.* $E = G \times G$, is a group. Throughout this work, one can see that all concepts (generalized subgroups, generalized normal subgroups, generalized homomorphisms, etc.) with respect to a generalized group $(G, E,\star)$ where E is the universal equivalence relation on G, coincide with the corresponding ones with respect to

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the classical concepts.

Often, if there is no confusion concerning E and $\star$, we denote $(G,E,\star)$ simply by stating G.

The following proposition says that from a collection of pairwise disjoint groups we can construct a generalized group. But, in the main, a generalized group is not just a union of a collection of pairwise disjoint groups, see 2.5. Example.

2.2. Proposition:

Assume that $\{(G_i,\star_i)\}_{i\in\Lambda}$ is a collection of pairwise disjoint groups (resp. semigroups), i.e. $G_i \cap G_j = \emptyset$ for any $i,j \in \Lambda$ with $i \neq j$ and E is an equivalence relation on $\cup_{i\in\Lambda} G$ defined by xEy iff $x,y \in G_i$ for some $i \in \Lambda$. If we define $\star$ on $\cup_{i\in\Lambda} G \times \cup_{i\in\Lambda} G$ by $x \star y := x \star_i y$ if $x,y \in G_i$ for some $i \in \Lambda$, and $x \star y := x$ if $x \in G_i, y \in G_j, i \neq j$ for some $i,j \in \Lambda$. Then, the triple $(\cup_{i\in\Lambda} G, E, \star)$ is a generalized group.

The following proposition shows that Molaei's generalization is a special case of the generalization detailed in this article.

2.3. Proposition:

Assume that $(G,\star)$ is a Molaei generalized group. Let E be an equivalence relation defined on G by xEy iff $e(x) = e(y)$ for any $x,y \in G$. Then, $(G,E,\star)$ is a generalized group.

In Adeniran *et al.* (2009), they show that an abelian Molaei generalized group is a group. This is not true in the generalization detailed in this article as shown in 2.8 Example.

2.4. Proposition:

For a generalized group G and $x \in G$, we have the following:

i. $[x] = [x^{-1}] = [e(x)]$.
ii. The identity element $e(x)$ of $[x]$ is unique.
iii. The inverse x^{-1} is unique.
iv. For any $x,y \in G$, $e(x) = e(y)$ iff $[x] = [y]$.
v. $e(e(x)) = e(x^{-1}) = e(x)$.

Proof. (i) It is clear, since $e(x), x^{-1} \in [x]$.

(ii) Suppose that $e_1(x)$ and $e_2(x)$ are both identities elements of x. Then, for $t \in [x]$, $t = te_1(x) = e_1(x)t$ and $t = te_2(x) = e_2(x)t$. Since, $e_1(x), e_2(x) \in [x]$ and $e_1(x), e_2(x)$ are identities of $[x]$, $e_1(x) = e_1(x)e_2(x) = e_2(x)e_1(x)$ and $e_2(x) = e_2(x)e_1(x)$ which implies that $e_1(x) = e_2(x)$.

(iii) Suppose x has two inverses $y,z \in [x]$. Then, $y = ye(y) = ye(x) = yxz = e(x)z = e(z)z = z$.

(iv) Let $e(x) = e(y)$. Since $e(x) \in [x]$ and $e(y) \in [y]$, $e(x) = e(y) \in [x] \cap [y]$ and hence $[x] = [y]$. Conversely, if $[x] = [y]$, then by uniqueness of the identity $e(x) = e(y)$.

(v) The proof is directly obtained from part (iv), since $e(x), x^{-1} \in [x]$.

In the following, we introduce some examples of generalized groups which are not groups.

2.5. Example:

Consider the set $\mathbb{Z}_{12} := \{0,1,2,\cdots,11\}$. Define the equivalence relation E on $\mathbb{Z}_{12}$ by rEs iff there exists $k \in \mathbb{Z}$ such that $r = s +_{12} 3k$ for any $r,s \in \mathbb{Z}_{12}$. The equivalence classes of this relation are $[0] = \{0,3,6,9\}, [1] = \{1,4,7,10\}, [2] = \{2,5,8,11\}$. Let $\overline{t} := \min[t]$ for $t \in \mathbb{Z}_{12}$ and $k_t := \frac{t-\overline{t}}{3}$ for any $t \in \mathbb{Z}_{12}$. Then, $\overline{0} = 0$, $\overline{1} = 1, \overline{2} = 2, \overline{3} = 0, \overline{4} = 1, \overline{5} = 2, \overline{6} = 0, \overline{7} = 1, \overline{8} = 2, \overline{9} = 0$, $\overline{10} = 1$, $\overline{11} = 2$ and $k_0 = 0, k_1 = 0, k_2 = 0, k_3 = 1, k_4 = 1, k_5 = 1, k_5 = 1, k_6 = 2, k_7 = 2, k_8 = 2, k_9 = 3, k_{10} = 3, k_{11} = 3$. Assume that the binary operation $\star$ on $\mathbb{Z}_{12}$ is defined by

$$r \star s = \overline{r} +_{12} 3(k_r + k_s) \ for \ r,s \in \mathbb{Z}_{12}.$$

From Table 1, the triple $(\mathbb{Z}_{12}, E, \star)$ is a generalized group with $\overline{t}$ the identity element of $[t]$ and $0^{-1} = 0, 1^{-1} = 1, 2^{-1} = 2, 3^{-1} = 9, 4^{-1} = 10, 5^{-1} = 11, 6^{-1} = 6, 7^{-1} = 7, 8^{-1} = 8, 9^{-1} = 3, 10^{-1} = 4, 11^{-1} = 5$. One can check easily that $(\mathbb{Z}_{12},\star)$ is not a group.

Table 1: The binary operation $\star$ on $\mathbb{Z}_{12}$.

$\star$	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	3	3	6	6	6	6	9	9	9
1	1	1	1	4	4	7	7	7	7	10	10	10
2	2	2	2	5	5	8	8	8	8	11	11	11
3	3	3	3	6	6	9	9	9	9	0	0	0
4	4	4	4	7	7	10	10	10	10	1	1	1
5	5	5	5	8	8	11	11	11	11	2	2	2
6	6	6	6	9	9	0	0	0	0	3	3	3
7	7	7	7	10	10	1	1	1	1	4	4	4
8	8	8	8	11	11	2	2	2	2	5	5	5
9	9	9	9	0	0	3	3	3	3	6	6	6
10	10	10	10	1	1	4	4	4	4	7	7	7
11	11	11	11	2	2	5	5	5	5	8	8	8

2.6. Example:

Let $\mathbb{R}$ be the set of all real numbers and fix $q \in \mathbb{R}$, $0 < q < 1$. Define E on $\mathbb{R}$ by xEy iff there is a $k \in \mathbb{Z}$ such that $x = q^k y$ for any $x,y \in \mathbb{R}$. One can easily show that E is an equivalence relation. Now, define $\star_q$ by

$$x \star_q y = q^{k_x+k_y}\overline{x} \ for \ x,y \in \mathbb{R},$$

where $\overline{x} := \max([x] \cap [0,1))$ and $k_x := \log_q(x/\overline{x})$ for any $x \in \mathbb{R}$. Clearly, the operation $\star_q$ is well defined. For any $x,y \in [t]$ and $t \in \mathbb{R}$, we can check that $\overline{(\overline{x})} = \overline{x} = t$, $\overline{q^{k_y}\overline{x}} = \overline{x}$, $\overline{q^{-k_y}\overline{x}} = \overline{x}$, $k_{\overline{x}} = 0$, $k_{q^{k_y}\overline{x}} = k_y$, and $k_{q^{-k_y}\overline{x}} = -k_y$. Also, we can show that for $[t] \subseteq \mathbb{R}$ and $x,y,z \in [t]$:

- $x \star_q y = q^{k_x+k_y}\overline{t} \in [t]$,
- $x \star_q (y \star_q z) = x \star_q \left(q^{k_y+k_z}\overline{y}\right) = q^{\left\{k_x+k_{\overline{q^{k_y+k_z}\overline{y}}}\right\}}\overline{x} = q^{k_x+k_y+k_z}\overline{x}$, and similarly $(x \star_q y) \star_q z = q^{k_x+k_y+k_z}\overline{x}$,
- $\overline{t} \star_q x = q^{\{k_{\overline{t}}+k_x\}}\overline{(\overline{t})} = q^{k_x}\left(\overline{t}\right) = x$, and similarly $x \star_q \overline{t} = x$.

Hence, the triple $(\mathbb{R}, E, \star_q)$ is a generalized group.

2.7. Example:

Fix $\omega \in \mathbb{R}, \omega > 0$. Assume the triple $(\mathbb{R}, E, \star_\omega)$ where E is defined by xEy iff there is $k \in \mathbb{Z}$ such that $x = y + k\omega$. Easily, one can show that E is an equivalence relation. The binary operation $\star_\omega$ is defined by $x \star_\omega y = \overline{x} + (k_x + k_y)\omega \ for \ x,y \in \mathbb{R}$,

where $\overline{t} := \min([t] \cap [0,\infty))$ and $k_t := \frac{t-\overline{t}}{\omega}$ for any $t \in \mathbb{R}$. The operation $\star_\omega$ is well defined. One can check for any $x,y \in [t]$ and $t \in \mathbb{R}$ that $\overline{(\overline{x})} = \overline{x} = t$, $\overline{\overline{x} + k_y\omega} = \overline{x}$, $\overline{\overline{x} - k_y\omega} = \overline{x}$, $k_{\overline{x}} = 0$, $k_{\overline{x}+k_y\omega} = k_y$, and $k_{\overline{x}-k_y\omega} = -k_y$. Now, for $[t] \subseteq \mathbb{R}$, we have the following:

- For any $x,y \in [t]$, $x \star_\omega y = \overline{t} + (k_x + k_y)\omega \in [t]$.
- For any $x,y,z \in [t]$, we have $x \star_\omega (y \star_\omega z) = x \star_\omega \left(\overline{y} + \left(k_y + k_z\right)\omega\right) = \overline{x} + \left(k_{(\overline{y}+(k_y+k_z)\omega)} + k_x\right)\omega = \overline{x} + \left(k_y + k_z + k_x\right)\omega$

and similarly $(x \star_\omega y) \star_\omega z = \overline{x} + \left(k_x + k_y + k_z\right)\omega$ which show that $\star_\omega$ is associative on $[t]$.

- For $x \in [t]$, $\overline{t} \star_\omega x = \overline{(\overline{t})} + \left(k_{\overline{t}} + k_x\right)\omega = \overline{t} + (0 + k_x)\omega = \overline{x} + k_x\omega = x$ and $x \star_\omega \overline{t} = \overline{x} + \left(k_x + k_{\overline{t}}\right)\omega = \overline{x} + k_x\omega = x$.

Therefore $\overline{t}$ is the identity element of $[t]$.

Hence, the triple $(\mathbb{R}, E, \star_\omega)$ is a generalized group.

2.8. Example:

Consider the set $\mathbb{Z}_6 := \{0,1,2,3,4,5\}$. Define the equivalence relation E on $\mathbb{Z}_6$ by rEs iff there exists $k \in \mathbb{Z}$ such that $r = s +_6 2k$ or $s = r +_6 2k$. Then clearly, the equivalence classes are $[0] = \{0,2,4\}$ and $[1] = \{1,3,5\}$. The binary operation $\star$ on $\mathbb{Z}_6$ is defined by Table 2. This triple $(\mathbb{Z}_6, E, \star)$ is an abeilan generalized group and

$(\mathbb{Z}_6, \star)$ is not a group.

Table 2: The binary operation $\star$ on $\mathbb{Z}_6$.

$\star$	0	1	2	3	4	5
0	0	0	2	0	4	0
1	0	1	0	3	0	5
2	2	0	4	0	0	0
3	0	3	0	5	0	1
4	4	0	0	0	2	0
5	0	5	0	1	0	3

3. Generalized Subgroups

In this section, we introduce and study the concept of a generalized subgroup.

3.1. Definition:

A nonempty subset H of a generalized group $(G, E, \star)$ is said to be a generalized subgroup if $(H, E, \star)$ is a generalized group.

The following proposition gives an easier criterion to decide whether a subset of a generalized group G is actually a generalized subgroup.

3.2. Proposition:

Assume that G is a generalized group and $\emptyset \neq H \subseteq G$. Then, H is a generalized subgroup if it satisfies the following conditions:

I. $x \star y \in H$ for any $x, y \in H$,
II. $e(x) \in H$ for any $x \in H$,
III. $x^{-1} \in H$.

3.3. Theorem:

Assume that G is a generalized group and $\emptyset \neq H \subseteq G$. Then, H is a generalized subgroup iff $x \star y^{-1} \in H$ for each $x, y \in H$.

Proof. For the ``only if" part: Let $H$ be a generalized subgroup and $x, y \in H$. Since $H$ is a generalized subgroup, $y^{-1} \in H$. Then, $x, y^{-1} \in H$ which implies that $x \star y^{-1} \in H$. For the ``if" part: Let us assume that $x \star y^{-1} \in H$ for each $x, y \in H$. Since $H \neq \phi, \exists t \in H$ and by assumption, we have $e(t) = t \star t^{-1} \in H$. If $y \in H$, then $e(y) \in H$ which implies that $y^{-1} = e(y) \star y^{-1} \in H$. Finally, for $x, y \in H$, $x, y^{-1} \in H$ and then $x \star y = x \star (y^{-1})^{-1} \in H$. In section 3.2. Proposition, H is a generalized subgroup.

3.4. Remark:

Let G be a generalized group. Then, the following statements hold:

i. G and $\{e(t)\}$ for each $t \in G$ are generalized subgroups of G. They are called trivial generalized subgroups of G.
ii. If $[t]$ is a generalized subgroup of G for each $t \in G$, then G is a union of pairwise disjoint generalized subgroups.
iii. If G is a group, then every generalized subgroup of G is a subgroup of G.
iv. The intersection of a family of generalized subgroups is necessarily a generalized subgroup.

3.5. Example:

Consider the generalized group $(\mathbb{R}, E, \star_q)$ which is defined in Example 2.6. Define the subsets H_1, H_2 as, $H_1 := \{x \in \mathbb{R}: k_x \in 2\mathbb{Z}\}$ and $H_2 := \{x \in \mathbb{R}: k_x \in 2\mathbb{Z} + 1\}$. We use Theorem 3.3 to show that H_1 is a generalized subgroup, but H_2 is not a generalized subgroup. Let $x, y \in H_1$ with $[x] = [y]$. Then, $\overline{x} = \overline{y}$ and $k_x, k_y \in 2\mathbb{Z}$. Note that $-k_y \in 2\mathbb{Z}$, and $k_x - k_y \in 2\mathbb{Z}$. Now, $x \star y^{-1} = q^{k_x - k_y}\overline{x} \in H_1$. Hence, H_1 is a generalized subgroup. However, H_2 is not a generalized subgroup, because $q^3x, q^5x \in H_2$ and $q^3x \star (q^5x)^{-1} = q^3x \star q^{-5}x = q^2x \notin H_2$.

4. Homomorphism of Generalized Groups

In this section, we give a definition of a homomorphism of generalized groups with some results.

4.1. Definition:

Assume that G and G' are generalized groups. A map $f: G \longrightarrow G'$ is called a *homomorphism* if for any $a, b \in G$, we have $f(ab) = f(a)f(b)$, and $[f(a)] = [f(b)]$ if $[a] = [b]$. The set $Ker_a(f) := \{x \in G: f(x) = e(a)\}$ is called the kernel of f with respect to $a \in G$.

The following theorem shows some of the basic results of the generalized groups homomorphisms.

4.2. Theorem:

Let G and G' be generalized groups and $f: G \longrightarrow G'$ be a homomorphism. Then:

i. $f(e(a)) = e(f(a))$.
ii. $f(a^{-1}) = (f(a))^{-1}$.
iii. $f(H)$ is a generalized subgroup of G' for any generalized subgroup H of G.
iv. $Ker_a(f)$ is a generalized subgroup of G for $a \in G$.
v. f is injective iff $Ker_a(f) = \{e(a)\}$ for all $a \in G$.

Proof. (i) Let $b = f(e(a))$ for $a \in G$. Then, $b = f(e(a)) = f(e(a)e(a)) = f(e(a))f(e(a)) = bb$ which implies that $e(b) = b$. That is, $e(f(a)) = f(e(a))$.

(ii) Let $b = a^{-1}$. Then, $e(f(a)) = f(e(a)) = f(aa^{-1}) = f(ab) = f(a)f(b)$ and $e(f(a)) = f(e(a)) = f(a^{-1}a) = f(a^{-1})f(a) = f(b)f(a)$ which show that $f(b)$ is the inverse of $f(a)$, completing the proof of (ii).

(iii) Let $K = f(H)$. It suffices to check that K is nonempty and $ab^{-1} \in K$ for any $a, b \in K$. Since $H \neq \phi$, there is $a \in H$ which implies that $f(a) \in K$; and so $K \neq \phi$. If $a, b \in K$, then $f^{-1}(a), f^{-1}(b) \in H$ and then $f^{-1}(a)(f^{-1}(b))^{-1} \in H$. Now, $f(f^{-1}(a)(f^{-1}(b))^{-1}) \in f(H) \Longrightarrow$ $f(f^{-1}(a))f((f^{-1}(b))^{-1}) \in K \Longrightarrow$ $f(f^{-1}(a))(f(f^{-1}(b)))^{-1} \in K \Longrightarrow ab^{-1} \in K$.

(iv) Let $a \in G$. Since $e(a) \in Ker_a(f)$, $Ker_a(f) \neq \phi$. Now, if $x, y \in Ker_a(f)$, then $f(x) = f(y) = f(e(a))$. We have

$$f(xy^{-1}) = f(x)f(y^{-1}) = f(e(a))(f(y))^{-1} = f(e(a))(f(e(a)))^{-1} = f(e(a))f(e(a)) = f(e(a)).$$

This implies that $xy^{-1} \in K$. Hence, K is a generalized subgroup.

(iv) Assume that f is injective. Then, for $a \in G$ there is at most one element that can be sent to the identity $e(f(a))$. Since $f(e(x)) = e(f(x))$, $Ker_a(f) = \{e(a)\}$. Conversely, let $Ker_a(f) = \{e(a)\}$ for all $a \in G$ and $f(x) = f(y)$ for some $x, y \in G$. Then, $f(x^{-1}y) = f(x^{-1})f(y) = f(x^{-1})f(x) = f(x^{-1}x) = f(e(x)) = e(f(x))$. This implies that $x^{-1}y \in Ker_x(f)$, and then $x^{-1}y = e(x)$. Thus, $x^{-1} = y^{-1}$, but the inverse is unique and so $x = y$. Therefore, f is injective.

Biography

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Dr. Aldwoah has been an assistant professor since 2009, of Yemeni nationality, and holder of a PhD from Cairo University, Egypt in 2009. He is interested in generalizing some mathematical concepts, time scales calculus, and dynamic equations. He worked as an assistant professor in some Yemeni and Saudi universities and is currently working at the Islamic University of Madinah. The author has attended five international conferences, and he has published many scientific papers in specialized scientific journals (ISI, Scopus).

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